Académie des sciences

Meeting on the International Year of Quantum Science and Technology
Exploring the quantum world

Étienne Ghys

One hundred and seventeen lines in 3-space

Yu. I. Manin, A Course in Mathematical Logic, Graduate Texts in Mathematics, 1977.

Garrett Birkhoff and John Von Neumann, The Logic of Quantum Mechanics, Annals of Mathematics, Vol. 37, No. 4 (Oct., 1936), pp. 823-843.

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Boolean Algebra :
Statements (true or false, 1 or 0), AND, OR, NOT, satisfying usual axioms, for instance

$$NOT(p\, OR\, q) = NOT(p)\, AND\, Not(q)$$

Partial boolean algebra : AND, OR, NOT, are partially defined.

Example : Orthogonal projections on subspaces of $\mathbb{R}^3$.
AND, OR defined when the subspaces are perpendicular, ie when projections commute.

Theorem : *this partial boolean algebra cannot be embedded in any full boolean algebra.*

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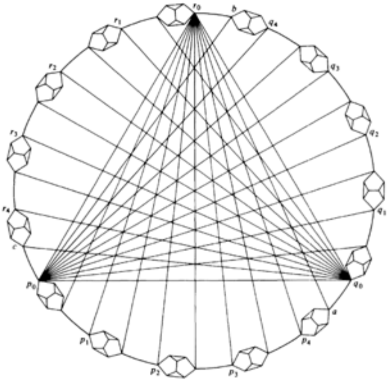
Proof : find 117 lines in $\mathbf{R}^3$, orthogonality according to this graph.



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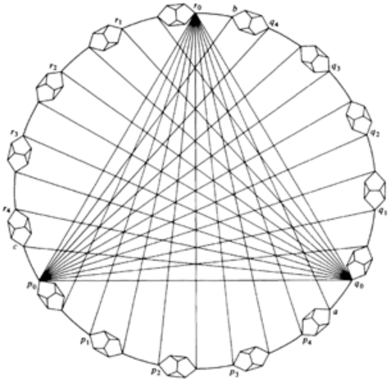

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